

Lecture Notes, Lecture 19

General Equilibrium in an Economy with unbounded technology sets: tossing the ball of radius c

Delete P.VI (bounded \mathcal{Y}^j). Like all good mathematicians, we're reducing this to the previous case.

Under assumptions of No Free Lunch (P.IV(a)) and Irreversibility (P.IV(b)), the attainable output set for the economy and for each firm is still bounded.

- P.IV. (a) if $y \in Y$ and $y \neq 0$, then $y_k < 0$ for some k .
(b) if $y \in Y$ and $y \neq 0$, then $-y \notin Y$

Let firm j 's (unbounded) production technology be Y^j . Define $S^j(p)$ as j 's profit maximizing supply in Y^j . Define $D^i(p)$ as i 's demand without restriction to $\{x \mid |x| \leq c\}$. Note that $S^j(p)$ and $D^i(p)$ may not be well defined.

Define $\tilde{Y}^j = Y^j \cap \{x \mid |x| \leq c\}$, substitute \tilde{Y}^j for Y^j in chapters 4 - 7. Define $\tilde{S}^j(p)$ as j 's supply function based on \tilde{Y}^j .

Theorem 8.3(b): If $\tilde{S}^j(p)$ exists and is attainable, then $S^j(p) = \tilde{S}^j(p)$.

Theorem 9.1(b): If $\tilde{D}^i(p)$ exists and is attainable, then $\tilde{D}^i(p) = D^i(p)$.

$$Z(p) = \sum_i D^i(p) - \sum_j S^j(p) - \sum_i r^i$$

Theorem 11.1: Assume P.II-P.V, and C.I-C.V, CVII, C.VIII. There is $p^* \in P$ so that p^* is an equilibrium price vector. That is, $Z(p^*) \leq 0$ and $p_k^* = 0$ for k so that $Z_k(p^*) < 0$.

Proof: The artificially bounded economy characterized by production technologies \tilde{Y}^j , $j \in F$, is a special case of the bounded economy of chapters 4 - 7. Find equilibrium of that bounded economy. That bounded economy equilibrium is attainable so restriction to length c is not a binding constraint. So bounded and unbounded supply and demand coincide. Equilibrium prices of the bounded

economy exist and are equilibrium prices for the unbounded economy with technology sets Y^j . Q.E.D.

Note that Theorem 11.1 does not require boundedness of technology sets and does not restrict households to make their consumption choices in a bounded set. Boundedness is used to establish the existence of market clearing prices, but at market clearing, the boundedness assumption can be deleted --- prices alone convey sufficient information to bound optimizing behavior.